Applied Statistics and Mathematics in Economics & Business

BS1501

Tutorial 3

Pairach Piboonrungroj
(Champ)

me@Pairach.com

1. A Yogurt manufacturer

(a) $$P(x > x_0) = P\left(z > \frac{x_0 - \mu}{\sigma_{\bar{x}}}\right)$$

$$x_0 = 570, \mu = 500, \sigma_{\bar{x}} = 33$$

$$P(x > 570) = P\left(z > \frac{570 - 500}{33}\right)$$

$$= P(Z > 2.12) = 0.017$$

1. A Yogurt manufacturer

(b)

$$P(x > x_0) = P\left(z > \frac{x_0 - \mu}{\sigma_{\bar{x}}}\right)$$

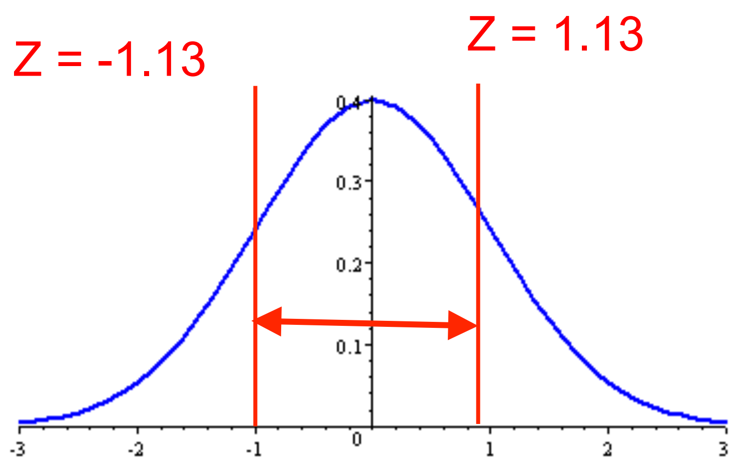
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{33}{\sqrt{14}} = 8.82$$

$$\boxed{\widetilde{x} \sim N(\mu = 500, \sigma_{\widetilde{x}} = 8.82)}$$

$$P(490 < \bar{x} < 510) = P\left(\frac{490-500}{8.82} < Z < \frac{510-500}{8.82}\right)$$

$$= P(-1.13 < Z < 1.13) = 1 - 2P(z > 1.13)$$

$$= 1 - 2(0.12924) = 0.740$$



Z = -1.13
Z = 1.13
0.4
0.3
0.2
0.1
-3
-2
-1
0
1
2
3

2. Women shift-workers

(a)

$$E(\hat{p}) = 0.60 \quad \sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{(0.60)(0.40)}{150}} = 0.04$$

$$P(0.60 < \hat{P} < 0.70) = P\left(\frac{0.60-0.60}{0.04} < Z < \frac{0.70-0.60}{0.04}\right)$$

$$P(0 < Z < 2.5) = 0.5 - 0.0062 = 0.4938$$

(b)

$$P(\hat{P} \geq 0.50) = P\left(Z > \frac{0.50-0.60}{0.04}\right)$$

$$P(Z > -2.5) = 1 - P(Z > 2.5) = 1 - 0.0062 = 0.9938$$

3. A Building Society

$$\boxed{\text{C.I. for } \mu = \bar{x} \pm Z_{\frac{\alpha}{2}} * \frac{s}{\sqrt{n}}}$$

(a) $$90\% \text{ C.I. for } \mu = \bar{x} \pm 1.645 * \frac{s}{\sqrt{n}}$$

$$= 15200 \pm 1.645 * \frac{8500}{\sqrt{30}}$$

$$\boxed{12667.16 < \mu < 17752.84}$$

(b) $$99\% \text{ C.I. for } \mu = \bar{x} \pm 2.57 * \frac{s}{\sqrt{n}} \quad = 15200 \pm 2.57 * \frac{8500}{\sqrt{30}}$$

$$\boxed{11211.67 < \mu < 19188.33}$$

4. Voters from the electorate

(a)

180 indicated their preference for a particular candidate (x)

400 sample voters (n)

$$\hat{P} = \frac{x}{n}$$

$$\hat{P} = \frac{180}{400} = 0.45$$

4. Voters from the electorate

(b)

$$\text{C.I. for } P = \hat{P} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{\mathrm{P}}(1-\hat{\mathrm{P}})}{\mathrm{n}}}$$

$$95\,\%\ \text{C.I. for } P = \hat{P} \pm Z_{\frac{0.05}{2}} \sqrt{\frac{\hat{\mathrm{P}}(1-\hat{\mathrm{P}})}{\mathrm{n}}}$$

$$= 0.45 \pm 1.96 \sqrt{\frac{0.45 * 0.55}{400}} = \quad 0.45 \pm 0.049$$

$$\boxed{0.401 < P < 0.499}$$

4. Voters from the electorate

(c)

$$\text{C.I. for } P = \hat{P} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{\mathrm{P}}(1-\hat{\mathrm{P}})}{\mathrm{n}}}$$

$$99\,\%\ \text{C.I. for } P = \hat{P} \pm Z_{\frac{0.01}{2}} \sqrt{\frac{\hat{\mathrm{P}}(1-\hat{\mathrm{P}})}{\mathrm{n}}}$$

$$= 0.45 \pm 2.57 \sqrt{\frac{0.45 * 0.55}{400}} = \quad 0.45 \pm 0.064$$

$$\boxed{0.386 < P < 0.514}$$

5. Lifetime of light bulbs

(a) $n = 100, \quad \bar{x} = 1570, \quad s = 120$

$H_0 : \mu = 1600 \qquad \alpha = 0.05 \Rightarrow$ **Reject H_0 if $z \leq -1.96$ or if $Z > +1.96$**

$H_1 : \mu \neq 1600 \qquad \alpha = 0.01$ **Reject H_0 if $z < -2.58$ or if $Z > +2.58$**

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1570 - 1600}{120/\sqrt{100}} = -2.5$$

$$\boxed{< -1.96 \text{ BUT } > -2.58}$$

$$\boxed{\therefore \text{Reject } H_0 \text{ at 5\% level of significance}}$$

i.e. There is sufficient evidence at 5% level of significance to reject the hypothesis that the mean lifetime of bulbs has changed, but not at the 1% level.

5. Lifetime of light bulbs

(b) $n = 100, \quad \bar{x} = 1570, \quad s = 120$

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1570 - 1600}{120/\sqrt{100}} = -2.5$$

$$H_0 : \mu = 1600$$

$$H_1 : \mu < 1600$$

$\alpha = 0.05 \quad \Rightarrow$ **Reject H_0 if $z < -1.64$ (one tailed)**

$\alpha = 0.01 \quad \Rightarrow$ **Reject H_0 if $z < -2.33$ (one tailed)**

$\therefore$ Reject H_0 5% and 1% levels of significance.

The first alternative hypothesis would seem to be a fairly weak one compared to the second one,
given that it would be of more interest to know whether the mean lifetime of the population of bulbs has deteriorated, rather than simply change.

6. Rent for student room in Cardiff

(a) $\begin{cases} H_0 : \mu = 220 \\ H_1 : \mu \neq 220 \end{cases}$

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{230 - 220}{35/\sqrt{40}}$$

=1.81

Reject H_0 if <u>Z<-1.96</u> or if Z>1.96.
Thus we do not reject H_0, i.e. the claim of £220 per month cannot be rejected.

(b) **C.I. for** $\mu = \bar{x} \pm Z_{0.025} * \frac{s}{\sqrt{n}} = 230 \pm 1.96 * \frac{35}{\sqrt{40}}$

$$219.15 < \mu < 240.85$$

Do not reject H_0 since £220 is in the above interval.

7. A bank manager & her customers

(a)

$$\begin{cases} H_0 : p = 0.30 \\ H_1 : p \neq 0.30 \end{cases}$$

$$\hat{p} = \frac{20}{100} = 0.20$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.30 * 0.70}{100}} = 0.045$$

$$Z = \frac{\hat{P} - P_0}{\sigma_{\hat{P}}} = \frac{0.20 - 0.30}{0.045} = -2.22 < -1.96$$

Therefore, we reject at 95% level of confidence.

(b)

Reject H_0 if Z<-2.57.

Thus do not reject H_0 at 1% level of significance.

Thank You for Your Attention

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