

Applied Statistics and Mathematics in Economics & Business

BS1501

Tutorial 2

Pairach Piboonrungroj
(Champ)

me@Pairach.com

1. Mutually Exclusive?

(a) Being an economist and being a professor:

Not mutually exclusive

(b) Throwing a 5 or 6 with one die:

Mutually exclusive

(c) Being an M.P. and being 20 years of age.
(Minimum age of an M.P. is 21):

Mutually exclusive

(d) Drawing a red card or an ace out of a pack of cards:

Not mutually exclusive

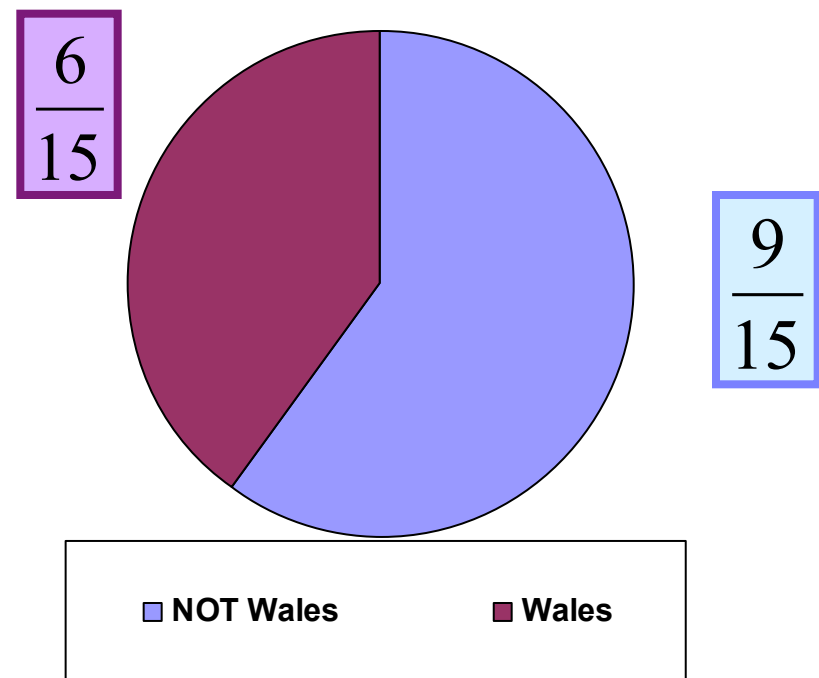
2.Accountancy employees

(a) Prob. of employees from **Wales**: $P(W)$
= from **Wales** / **All employees**
= **6/15**

(b) Prob. of employees **Scotland**: $P(S)$
= from **Scotland** / **All employees**
= **4/15**

2.Accountancy employees

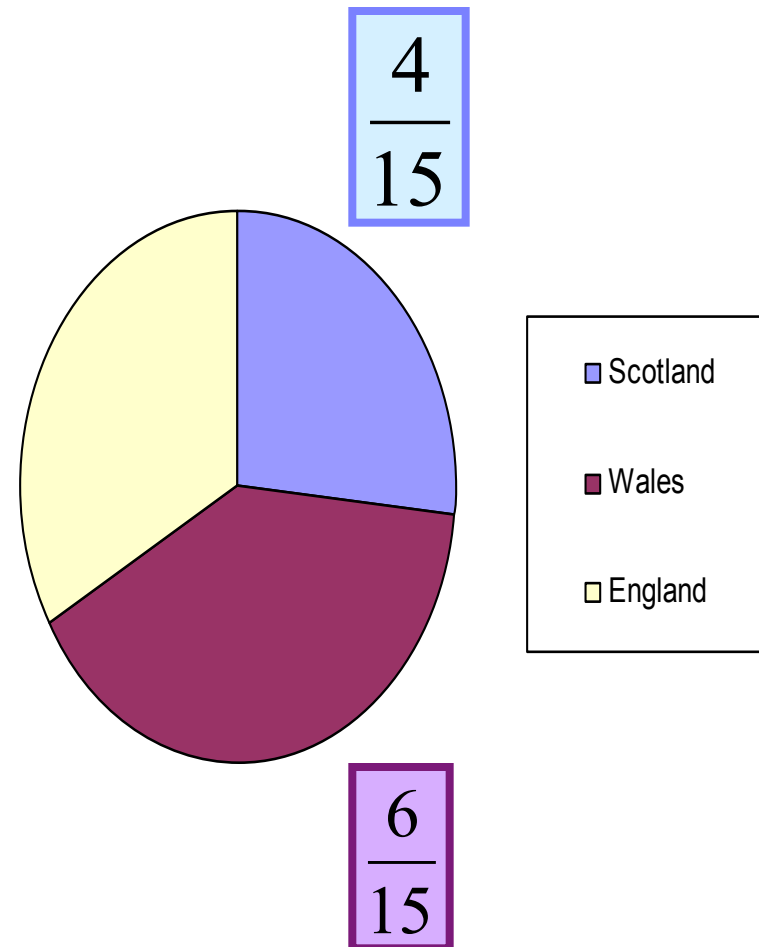
(c) Prob. of
employees **NOT**
from Wales:
=All prob. – $P(W)$
= $1 - P(W)$
= $1 - (6/15) = 9/15$



2.Accountancy employees

(d) Prob. of
employees
From **Wales** and
Scotland

$$\begin{aligned} &= P(W) + P(S) \\ &= (6/15) + (4/15) \\ &= \mathbf{10/15} \end{aligned}$$

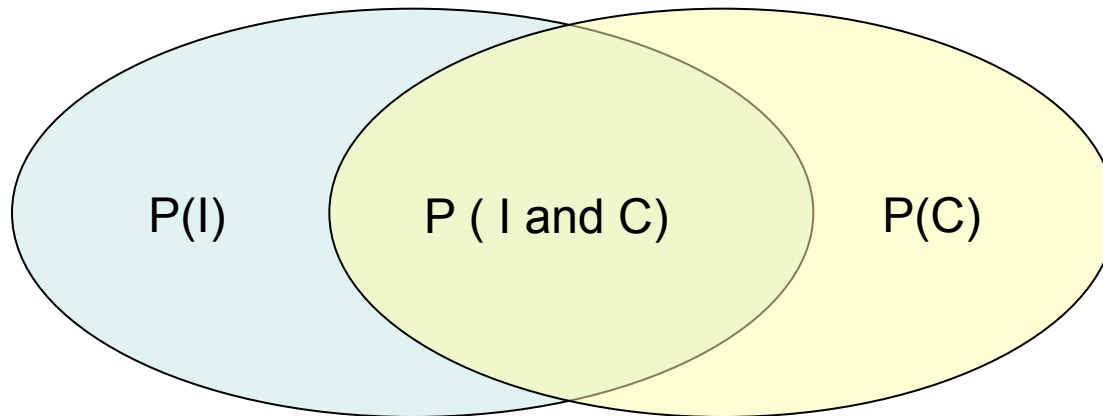


3. Primary school in Cardiff

I = the event that the boy likes ice-cream

C = the event that the boy likes cake

$$P(I) = 0.75, P(C) = 0.55, P(I \text{ and } C) = 0.40$$



$$\begin{aligned} \text{(a) } P(I \text{ or } C \text{ or both}) &= P(I) + P(C) - P(I \text{ and } C) \\ &= 0.75 + 0.55 - 0.40 = 0.90 \end{aligned}$$

3. Primary school in Cardiff

I = the event that the boy likes ice-cream

C = the event that the boy likes cake

$$P(I) = 0.75, P(C) = 0.55, P(I \text{ and } C) = 0.40$$

(b) Neither ice-cream nor cake

$$= 1 - P(I \text{ or } C \text{ or Both})$$

$$= 1 - 0.90$$

$$= 0.1$$

4. Accidents

	Accident	No accident
Under 45 years	25	35
45 years or older	5	35

4. Accidents

	Accident	No accident	Total
Under 45 years	25	35	60 (25 + 35)
45 years or older	5	35	40 (5 + 35)
Total	30 (25 + 5)	70 (35 + 35)	100

4. Accidents

(a)

	Accident	No accident	Total
Under 45 years	0.25 (25/100)	0.35 (35/100)	0.60 (60/100)
45 years or older	0.05 (5/100)	0.35 (35/100)	0.40 (40/100)
Total	0.30 (30/100)	0.70 (70/100)	1.0 (100/100)

4. Accidents

(b) $P(\text{Accident} \mid \text{under 45})$

$$= P(\text{Accident and under 45}) / P(\text{Under 45})$$

$$= 0.25 / 0.60 = 0.416$$

	Accident	No accident	Total
Under 45 years	0.25 (25/100)	0.35 (35/100)	0.60 ([25+35]/100)
45 years or older	0.05 (5/100)	0.35 (35/100)	0.40 (40/100)
Total	0.30 (30/100)	0.70 (70/100)	1.0 (100/100)

4. Accidents

(c) $P(\text{Accident} \mid \text{over 45})$

$$= P(\text{Accident and over 45}) / P(\text{over 45})$$

$$= 0.05 / 0.40 = 0.125$$

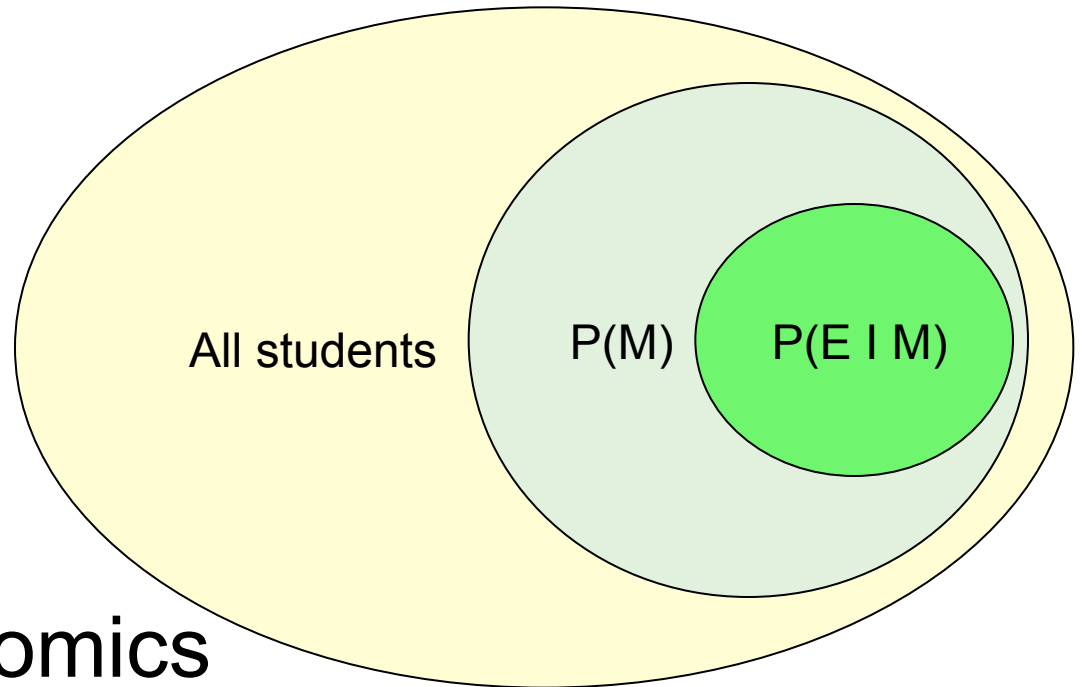
	Accident	No accident	Total
Under 45 years	0.25 (25/100)	0.35 (35/100)	0.60 ([25+35]/100)
45 years or older	0.05 (5/100)	0.35 (35/100)	0.40 (40/100)
Total	0.30 (30/100)	0.70 (70/100)	1.0 (100/100)

5. Economics course

$$P(M) = P(F) = 1/2$$

$$P(E | M) = 1/6$$

$$P(E | F) = 1/30$$



(a) Male and Economics

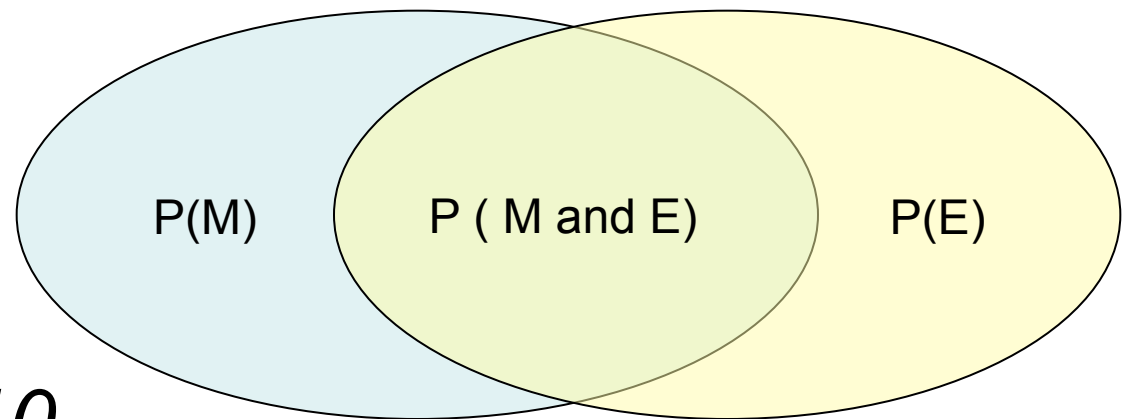
$$\begin{aligned} P(M \text{ and } E) &= P(E | M) * P(M) \\ &= (1/6) * (1/2) = 1/12 \end{aligned}$$

5. Economics course

$$P(M) = P(F) = 1/2$$

$$P(E | M) = 1/6$$

$$P(E | F) = 1/30$$



$$P(E) = 10\% = 1/10$$

$$\begin{aligned} \text{(b) } P(\mathbf{M \text{ or } E}) &= P(M) + P(E) - P(M \text{ and } E) \\ &= (1/2) + (1/10) - \mathbf{(1/12)} \\ &= \mathbf{31/60} \end{aligned}$$

6. Financial advisor

Combination:

Number of ways of selecting x object from a set of n, when the order is unimportant

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

$$n = 10$$

$$x = 4$$

$$(n-x) = 6$$

$$\begin{aligned}\binom{10}{4} &= \frac{10!}{4!6!} = \frac{10 * 9 * 8 * 7 * 6!}{4!6!} \\ &= \frac{10 * 9 * 8 * 7}{(4 * 3 * 2 * 1)} \\ &= \frac{10 * (3 * 3) * 8 * 7}{(8 * 3)} = \frac{10 * 3 * 7}{1} = 210\end{aligned}$$

7. Company' s Demand

Weekly unit Demand (x)	Probability: f(x)	E (x) = x*f(x)	x – E (x)	[x-E(x)]^2*f(x)
800	0.10	80.00	-185.00	3422.50
900	0.25	225.00	-85.00	1806.25
1000	0.40	400.00	15.00	90.00
1100	0.20	220.00	115.00	2645.00
1200	0.05	60.00	215.00	2311.25
Sum	Expected value	985.00	Variance	10275.00
			Std Deviation	101.37

7. Company' s Demand

$$\begin{aligned} \text{(a) } E(x) &= \text{Sum } [x \cdot f(x)] \\ &= 800(0.10) + 900(0.25) + \\ &\quad 1000(0.40) + 1100(0.20) + 1200(0.05) \\ &= 985 \end{aligned}$$

$$\begin{aligned} \text{(b) } \text{Var}(x) &= \text{Sum } [(x - E(x))^2 \cdot f(x)] \\ &= (800 - 985)^2(0.10) + (900 - 985)^2(0.25) \\ &\quad + (1000 - 985)^2(0.40) + (1100 - 985)^2(0.20) \\ &\quad + (1200 - 985)^2(0.05) \\ &= 10,275 \end{aligned}$$

$$\text{Standard deviation} = \sqrt{10,275} = 101.36$$

8. Insurance Salesperson

$$P(x) = \binom{n}{x} [p(x)]^x [1 - p(x)]^{n-x}$$

$$(a) \quad P(x = 6) = \binom{6}{6} (2/3)^6 (1/3)^0 = 0.088$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

$$(b) \quad P(x = 4) = \binom{6}{4} (2/3)^4 (1/3)^2 = 0.329$$

8. Insurance Salesperson

$$P(x) = \binom{n}{x} [p(x)]^x [1 - p(x)]^{n-x}$$
$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

$$(c) P(x > 1) = 1 - [P(x = 0) + P(x = 1)]$$

$$P(x = 0) = \binom{6}{0} (2/3)^0 (1/3)^6$$

$$P(x = 1) = \binom{6}{1} (2/3)^1 (1/3)^5$$

$$= 1 - 0.02$$

$$= 0.98$$

$$(d) \mu = np = 6(2/3) = 4$$

9. Fatal Rate

Poisson Distribution:

Estimating the number of occurrence of an event within specific interval of time

A) No death in a one year period
 $x = 0$

$$\lambda(1\text{year}) = 1.5$$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(x = 0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{1.5^0 e^{-1.5}}{0!} = 0.2231$$

9. Fatal Rate

Poisson Distribution:

Estimating the number of occurrence of an event within specific interval of time

b) No death in 6 months period

$$x = 0$$

$$\lambda(6months) = 1.5 / 2 = 0.75$$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(x = 0) = \frac{0.75^0 e^{-0.75}}{0!} = 0.4724$$

9. Fatal Rate

c) $P(\geq 2 \text{ accidents in 6 months})$

$$\lambda(6months) = 0.75$$

$$= 1 - P(x=0) - P(x=1)$$

$$= 1 - 0.4724 - 0.3543$$

$$= \mathbf{0.1733}$$

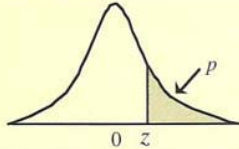
10. Train (Cardiff – London)

(a) $x \sim N(\mu = 120, \sigma = 10)$ $P(x > x_0) = p\left(Z > \frac{x_0 - \mu}{\sigma}\right)$

$$P(x > 130) = p\left(Z \geq \frac{130 - 120}{10}\right)$$

$$= P(Z > +1)$$

$$= 0.1586$$



z	Second decimal place of z									
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026

10. Train

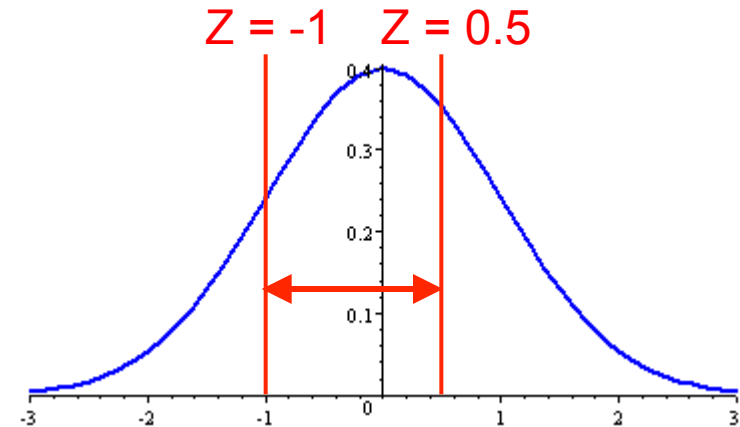
$$(b) \quad x \sim N(\mu = 120, \sigma = 10) \quad P(x > x_0) = p\left(Z \geq \frac{x_0 - \mu}{\sigma}\right)$$

$$P(110 < x < 125) = P\left(\frac{110 - 120}{10} < Z < \frac{125 - 120}{10}\right)$$

$$= P(-1 < Z < +0.5)$$

10. Train

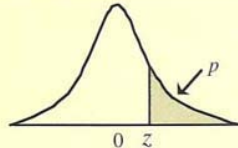
(b) $P(-1 < Z < +0.5)$



$$= 1 - [P(Z < -1) + P(Z > 0.5)]$$

$$P(Z < -1) = P(Z > 1)$$

$$= 1 - [P(Z > 1) + P(Z > 0.5)]$$



		Second decimal place of z									
z		.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0		.3900	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1		.4002	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2		.4007	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3		.3921	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4		.3445	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5		.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6		.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7		.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8		.2118	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9		.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0		.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1		.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2		.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3		.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4		.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5		.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6		.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7		.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8		.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9		.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0		.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1		.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2		.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3		.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4		.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5		.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6		.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7		.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026

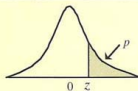
$$= 1 - 0.15866 - 0.30854$$

$$= 0.5328$$

10. Train

(c) $x \sim N(\mu = 120, \sigma = 10)$

$$P(x > x_0) = p\left(Z \geq \frac{x_0 - \mu}{\sigma}\right)$$



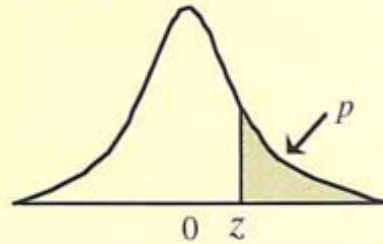
Standard normal distribution table showing the area to the right of z (p) for various values of z. The table is organized with z values in the first column and the second decimal place of z in the top row.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0969	.0951	.0934	.0916	.0897	.0881	.0863	.0846	.0829	.0813
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026

$$P(x > x_0) = p\left(Z \geq \frac{x_0 - 120}{10}\right)$$

$P(x > x_0) = 10\% = 0.10$

$$0.10 = p\left(Z \geq \frac{x_0 - 120}{10}\right)$$



Second decimal place of z										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026

$$0.10 = p\left(Z \geq \frac{x_0 - 120}{10}\right)$$

$$1.2816 = \left(\frac{x_0 - 120}{10}\right)$$

$$X_0 = 132.816$$

minutes

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me@pairach.com